Appropriate Reasoning about Data Dependencies in Fixed and Undetermined Universes

— On Learning Theories of Functional and Multivalued Dependencies —

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Outline

Motivation: Efficient Updates and Querying

Criticism: Desirable Properties of Inference Systems

Some Learning Inference Systems for FDs and MVDs

Appropriate Reasoning in Fixed Universes

An Axiomatisation in Undetermined Universes

Extensions to Full Hierarchical Dependencies
Motivation: A Design Problem

• manufacturers supply articles from a location at a certain cost
• Article, Manufacturer, Location, and Costs
• set $\Sigma$ of semantic constraints:
  • Article $\rightarrow$ Manufacturer
  • Article, Location $\rightarrow$ Costs
  • Manufacturer $\rightarrow$ Location
• target database is supposed to
  • process efficiently updates on Costs based on the Article, e.g.,
    $\pi_{\text{Costs}}(\sigma_{\text{Article} = \text{MP3-Player}}(\{\text{Article}, \text{Costs}\}))$
  • efficiently process queries about Article, Location-information of manufacturers such as
    $\pi_{\text{Article, Location}}(\sigma_{\text{Manufacturer} = \text{Sony}}(\{\text{Article}, \text{Manufacturer}, \text{Location}\}))$
• good design that meets these criteria?
Motivation: A Design Solution

- the FD $Article \rightarrow Costs$ is implied by $\Sigma$
- as reasonable semantic constraint it results in decomposition
  $\{Article, Costs\}$ and $\{Article, Manufacturer, Location\}$
- efficient updates on first schema (no data redundancy)
- efficient query processing on second schema (no joining necessary)
- efficient solutions to implication problem unlock good design choices
- inference systems to represent properties of constraints appropriately
Reminder of Definitions

• FD $X \rightarrow Y$ on relation schema $R$ satisfied by $R$-relation $r$ iff for all $t_1, t_2 \in r$: if $t_1[X] = t_2[X]$, then $t_1[Y] = t_2[Y]$

• MVD $X \rightarrow\rightarrow Y$ on relation schema $R$ satisfied by $R$-relation $r$ iff for all $t_1, t_2 \in r$: if $t_1[X] = t_2[X]$, then there is some $t \in r$ such that $t[XY] = t_1[XY]$ and $t[X(R - XY)] = t_2[X(R - XY)]$

• if $r$ satisfies $X \rightarrow Y$, then $r = r[XY] \bowtie r[X(R - XY)]$

• $r$ satisfies $X \rightarrow\rightarrow Y$ if and only if $r = r[XY] \bowtie r[X(R - XY)]$

• $\Sigma \cup \{\varphi\}$ set of FDs and MVDs on $R$: $\Sigma$ $R$-implies $\varphi$ iff every $R$-relation $r$ that satisfies all $\sigma \in \Sigma$ also satisfies $\varphi$
Example

• consider relation $r$ over \{Article, Manufacturer, Location, Costs\}:

<table>
<thead>
<tr>
<th>Article</th>
<th>Manufacturer</th>
<th>Location</th>
<th>Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP3-Player</td>
<td>Sony</td>
<td>Singapore</td>
<td>500</td>
</tr>
<tr>
<td>Camera</td>
<td>Sony</td>
<td>Hong Kong</td>
<td>800</td>
</tr>
<tr>
<td>Camera</td>
<td>Sony</td>
<td>Singapore</td>
<td>800</td>
</tr>
<tr>
<td>MP3-Player</td>
<td>Sony</td>
<td>Hong Kong</td>
<td>500</td>
</tr>
</tbody>
</table>

• $r$ violates $Manufacturer \rightarrow Location$

• $r$ satisfies $Manufacturer \rightarrow Location$

• $r$ is the lossless join of the following two projections:

<table>
<thead>
<tr>
<th>Manufacturer</th>
<th>Location</th>
<th>Article</th>
<th>Manufacturer</th>
<th>Costs</th>
</tr>
</thead>
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<td>Sony</td>
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<td>Hong Kong</td>
<td>Camera</td>
<td>Sony</td>
<td>800</td>
</tr>
</tbody>
</table>
Sound Inference Rules under Discussion

\[
\frac{X \rightarrow Y}{Y \subseteq X} \quad \frac{X \rightarrow Y}{X \rightarrow XY} \quad \frac{X \rightarrow Y, Y \rightarrow Z}{X \rightarrow Z} \\
\text{(reflexivity, } R_F) \quad \text{(extension, } E_F) \quad \text{(transitivity, } T_F)
\]

\[
\frac{X \rightarrow Y}{X \rightarrow Y \subseteq U} \quad \frac{X \rightarrow Y}{X \rightarrow R - Y} \quad \frac{X \rightarrow Y, Y \rightarrow Z}{X \rightarrow Z - Y} \\
\text{(augmentation, } A_M) \quad \text{(R-complementation, } C^R) \quad \text{(pseudo-transitivity, } T_M)
\]

\[
\frac{X \rightarrow Y, W \rightarrow Z}{X \rightarrow Y \cap Z} \quad \frac{X \rightarrow Y, Y \rightarrow Z}{X \rightarrow YZ} \quad \frac{X \rightarrow Y}{X \rightarrow Y} \\
\text{Y } \cap \text{ W } = \emptyset \quad \text{(subset, } S_M) \quad \text{(additive transitivity, } T^*_M) \quad \text{(implication, } I_{FM})
\]

\[
\frac{X \rightarrow Y, Y \rightarrow Z}{X \rightarrow Z - Y} \quad \frac{X \rightarrow Y, W \rightarrow Z}{X \rightarrow Y \cap Z} \\
\text{(mixed pseudo-transitivity, } T_{FM}) \quad \text{(mixed subset, } S_{FM})
\]
Complete, but not Complementary

• $\mathcal{F}_C = \{ R_F, E_F, T_F, A_M, T_M, I_{FM}, T_{FM}, C_R^M \}$ is $R$-complete for all $R$
• $R = ABCD$ with $A = \text{Movie}$, $B = \text{Actor}$, $C = \text{Title}$, $D = \text{YearBorn}$
• inference of $A \rightarrow B, C$ from $\Sigma = \{ A \rightarrow B, A \rightarrow C \}$ using $\mathcal{F}_C$

$A \rightarrow C$

$\vdash A \rightarrow B$

$\frac{A \rightarrow A, B}{A \rightarrow A, B} A_M$

$\frac{A \rightarrow A, B, D}{A \rightarrow A, B, D} C_R^M$

$\frac{A, B \rightarrow A, B, D}{A, B \rightarrow A, B, D} A_M$

$\frac{A \rightarrow D}{A \rightarrow D} T_M$

$\frac{A \rightarrow A, D}{A \rightarrow A, D} A_M$

$\frac{A \rightarrow B, C}{A \rightarrow B, C} C_R^M$

• some MVD-inferences by $\mathcal{F}_C$ require $C_R^M$ not just in last step
Complementarity

• Biskup’80: $\mathcal{C}_C = \mathcal{F}_C \cup \{S_M, T_M^*\}$ is $R$-complementary for all $R$

• $R = ABCD$ with $A = \text{Movie}$, $B = \text{Actor}$, $C = \text{Title}$, $D = \text{YearBorn}$

• inference of $A \rightarrow B, C$ from $\Sigma = \{A \rightarrow B, A \rightarrow C\}$ using $\mathcal{C}_C$

\[
\begin{align*}
A \rightarrow A & \quad \mathcal{R}_F \\
A \rightarrow A & \quad \mathcal{I}_{FM} \\
A \rightarrow A, B & \quad \mathcal{A}_M \\
A, B \rightarrow C & \quad \mathcal{T}_M^* \\
A \rightarrow A, B, C & \quad \mathcal{T}_M \\
A \rightarrow B, C & \quad \mathcal{T}_M
\end{align*}
\]

• every MVD-inference by $\mathcal{C}_C$ requires $C^R_M$ in at most last step
Complete and Complementary, but not Adequate

• \( \mathcal{C}_C = \{ \mathcal{R}_F, \mathcal{E}_F, \mathcal{T}_F, A_M, T_M, S_M, T_M^*, I_{FM}, T_{FM}, C_M^R \} \)

• \( R = ABC \):
  inference of \( A \to B \) from \( \Sigma = \{ A \to B, C \to B \} \) using \( \mathcal{C}_C \)

\[
\begin{align*}
A \to B \\
\cline{2-2}
\frac{A \to A, B A_M}{A \to A, B A_M} \\
\frac{A \to C C_M^R}{A \to C C_M^R} \\
\frac{C \to B T_{FM}}{A \to B T_{FM}}
\end{align*}
\]

• some FD-inferences by \( \mathcal{C}_C \) require \( C_M^R \)
Appropriate: Complementary and Adequate

• \( \mathcal{AC}_c = (\mathcal{C}_c - \{T_{FM}\}) \cup \{S_{FM}\} \)

• \( R = ABC \):
  inference of \( A \rightarrow B \) from \( \Sigma = \{A \rightarrow B, C \rightarrow B\} \) using \( \mathcal{AC}_c \)
  \[
  \begin{array}{ccc}
  A & \rightarrow & B \\
  & & C \rightarrow B \\
  \hline
  A & \rightarrow & C
  \end{array}
  \]

• no FD-inference by \( \mathcal{AC}_c \) requires \( C_R \)
Definitions of Complementarity and Adequacy

- let $\mathcal{S}$ contain some inference rules under discussion and $C_M^R$

- Biskup’80:
  $R$-complete set $\mathcal{S}$ is said to be $R$-complementary if and only if for every set $\Sigma$ of FDs and MVDs on $R$ and every MVD $\varphi$ on $R$ such that $\varphi$ is $R$-implied by $\Sigma$ there is an inference of $\varphi$ from $\Sigma$ by $\mathcal{S}$ in which the $R$-complementation rule $C_M^R$ is applied at most once, and if it is applied, then it is applied in the last step of the inference.

- Biskup/Link’08:
  $R$-complete set $\mathcal{S}$ is said to be $R$-adequate if and only if for every set $\Sigma$ of FDs and MVDs on $R$ and every FD $\varphi$ on $R$ such that $\varphi$ is $R$-implied by $\Sigma$ there is an inference of $\varphi$ from $\Sigma$ by $\mathcal{S}$, i.e., an inference in which the $R$-complementation rule $C_M^R$ is not utilised at all.
Main Results for FDs and MVDs

\[ \mathcal{A}_C = \mathcal{A}_C \cup \{S_M, T_M^*\} = (C_C - \{T_{FM}\}) \cup \{S_{FM}\} \]

appropriate: adequate and complementary

\[ \mathcal{A}_C = (\mathcal{F}_C - \{T_{FM}\}) \cup \{S_{FM}\} \]

adequate

\[ \mathcal{C}_C = \mathcal{F}_C \cup \{S_M, T_M^*\} \]

complementary

\[ \mathcal{F}_C = \{R_F, E_F, T_F, A_M, T_M, I_{FM}, T_{FM}, C^R_M\} \]
Inadequacy of $\mathbf{C}_C$ and $\mathcal{F}_C$

- $\mathbf{C} = \{R_F, \mathcal{E}_F, \mathcal{T}_F, \mathcal{A}_M, \mathcal{T}_M, \mathcal{S}_M, \mathcal{T}_M^*, \mathcal{I}_{FM}, \mathcal{I}_{FM}\}$
- $\Sigma = \{\emptyset \rightarrow A, B \rightarrow A\}$ and $\varphi = \emptyset \rightarrow A$
- closure of $\Sigma$ under inferences with $\mathbf{C}$ (no trivial FDs/MVDs):

<table>
<thead>
<tr>
<th></th>
<th>$\emptyset$</th>
<th>$A$</th>
<th>$B$</th>
<th>$AB$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\emptyset$</td>
<td>$\times$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A$</td>
<td>$\times$</td>
<td>$\times$</td>
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</tr>
<tr>
<td>$B$</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$\times$</td>
</tr>
<tr>
<td>$AB$</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$\times$</td>
</tr>
</tbody>
</table>

- $\varphi \notin \Sigma_C^+$ but $\varphi \in \Sigma_C^+ \cup \{\mathcal{S}_{FM}\}$
- there is a relation schema $R$, a set $\Sigma$ of FDs and MVDs on $R$ and an FD $\varphi$ on $R$ such that $\varphi \in \Sigma_C^+$ but $\varphi \notin \Sigma_C^+$
- systems $\mathcal{F}_C$ and $\mathbf{C}_C$ are not $R$-adequate for some $R$
Appropriateness of $\mathcal{AC}$

• $\mathcal{AC} = \{ R_F, E_F, T_F, A_M, T_M, T_{FM}, S_{FM}, C^R_M \}$ is $R$-complete for all $R$

• Let $R$ be some relation schema, and let $\Sigma$ be a set of FDs and MVDs on $R$. For every inference $\gamma$ from $\Sigma$ by the system $\mathcal{AC}$ there is an inference $\xi$ from $\Sigma$ by the system $\mathcal{AC}_C = \mathcal{AC} \cup \{ S_M, T^*_M \}$ with:

(i) if $\gamma$ infers an MVD, then
• $\gamma$ and $\xi$ infer the same MVD,
• in $\xi$ the $R$-complementation rule $C^R_M$ is applied at most once, and
• if $C^R_M$ is applied in $\xi$, then $C^R_M$ is applied as the last rule.

(ii) if $\gamma$ infers an FD, then
• $\gamma$ and $\xi$ infer the same FD, and
• in $\xi$ the $R$-complementation rule $C^R_M$ is not applied at all.
Example Transformation into an Appropriate Inference

• inappropriate inference step:

\[
\begin{align*}
X & \rightarrow Y \\
C_M^R: & \quad X \rightarrow R - Y \\
S_{FM}: & \quad X \rightarrow (R - Y) \cap Z \\
& \quad = Z - Y \\
W & \rightarrow Z
\end{align*}
\]

\[(R - Y) \cap W = \emptyset.\]

• since \((R - Y) \cap W = \emptyset\) holds we have \(W \subseteq Y\); replacement:

\[
\begin{align*}
\mathcal{R}_F: & \quad Y \rightarrow W^{W \subseteq Y} \quad W \rightarrow Z \\
\mathcal{T}_F: & \quad Y \rightarrow Z \\
X & \rightarrow Y \\
\mathcal{T}_{FM}: & \quad Y \rightarrow Z \\
\mathcal{T}_M: & \quad X \rightarrow Z - Y \\
\mathcal{T}_{FM}: & \quad X \rightarrow (Z - Y) \cap Z \\
& \quad = Z - Y
\end{align*}
\]
Nearly Complete Reasoning in Fixed Universes

\[ \mathcal{AC} = \{ R_F, E_F, T_F, A_M, T_M, S_M, T_M^*, I_{FM}, S_{FM} \} \]

Let \( \Sigma \cup \{ \varphi \} \) be a finite set of FDs and MVDs with \( \bigcup_{\sigma \in \Sigma} \text{Attr}(\sigma) \cup \text{Attr}(\varphi) \subseteq R \). Then

- If \( \varphi \) denotes an FD, then: \( \varphi \in \Sigma_{\mathcal{AC}}^+ \) if and only if \( \varphi \in \Sigma_{\mathcal{AC}}^+ \).

- If \( \varphi \) denotes the MVD \( X \rightarrow Y \), then: \( X \rightarrow Y \in \Sigma_{\mathcal{AC}}^+ \) if and only if \( X \rightarrow Y \in \Sigma_{\mathcal{AC}}^+ \) or \( X \rightarrow (R - Y) \in \Sigma_{\mathcal{AC}}^+ \).
Implication in Undetermined Universes

- Let $\Sigma \cup \{\varphi\}$ be a set of FDs and MVDs. We say that $\Sigma$ implies $\varphi$ if and only if every relation $r$ satisfies the following condition: if $\bigcup_{\sigma \in \Sigma} \text{Attr}(\sigma) \cup \text{Attr}(\varphi) \subseteq \text{Dom}(r)$ and $r$ satisfies all $\sigma \in \Sigma$, then $r$ also satisfies $\varphi$.

- $\text{FILM} = \{\text{Movie, Director, Actor}\}$ and
  \[
  \Sigma = \{\text{Movie} \rightarrow \text{Director}, \text{Actor} \rightarrow \text{Director}\}
  \]

- $\Sigma$ FILM-implies $\text{Movie} \rightarrow \text{Actor}$ but does not imply $\text{Movie} \rightarrow \text{Actor}$

<table>
<thead>
<tr>
<th>Movie</th>
<th>Director</th>
<th>Actor</th>
<th>Year_born</th>
</tr>
</thead>
<tbody>
<tr>
<td>La Dolce Vita</td>
<td>Frederico Fellini</td>
<td>M. Mastroianni</td>
<td>1924</td>
</tr>
<tr>
<td>La Dolce Vita</td>
<td>Frederico Fellini</td>
<td>A. Ekberg</td>
<td>1931</td>
</tr>
</tbody>
</table>

- Let $\Sigma$ be a set of functional dependencies. Then $\Sigma$ implies the MVD $X \rightarrow Y$ if and only if $\Sigma$ implies the FD $X \rightarrow Y$. 

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First ever combined Axiomatisation of FDs and MVDs in Undetermined Universes

- $\mathcal{AC} = \{ \mathcal{R}_F, \mathcal{E}_F, \mathcal{I}_F, \mathcal{A}_M, \mathcal{T}_M, \mathcal{S}_M, \mathcal{T}_M^*, \mathcal{I}_{FM}, \mathcal{S}_{FM} \}$ is sound and complete for FD and MVD implication in undetermined universes

- completeness of $\mathcal{AC}$: assume $\varphi \notin \Sigma^+_\mathcal{AC}$
  - $T := \bigcup_{\sigma \in \Sigma} \text{Attr}(\sigma) \cup \text{Attr}(\varphi) \subseteq \text{Dom}(r)$
  - let $R \subseteq A$ be finite proper superset of $R$ $(R - \text{rhs}(\varphi) \not\subseteq T)$
  - if $\varphi$ FD, then $\varphi \notin \Sigma^+_\mathcal{AC}$, but $R$-completeness of $\mathcal{AC}_C$ shows that $\Sigma$ does not $R$-imply $\varphi$; hence, $\Sigma$ does not imply $\varphi$
  - if $\varphi = X \rightarrow Y$, then $X \rightarrow R - Y \notin \Sigma^+_\mathcal{AC}$ since $R - Y$ is not a subset of $T$; from $X \rightarrow Y \notin \Sigma^+_\mathcal{AC}$ and $X \rightarrow R - Y \notin \Sigma^+_\mathcal{AC}$ we conclude $X \rightarrow Y \notin \Sigma^+_\mathcal{AC}_C$; $R$-completeness of $\mathcal{AC}_C$ shows that $\Sigma$ does not $R$-imply $\varphi$; hence, $\Sigma$ does not imply $\varphi$
Full Hierarchical Dependencies

- **Full hierarchical dependency** (FHD) on relation schema $R$ is expression $X : S$ where $X \subseteq R$ and $S$ is a non-empty set of pairwise disjoint subsets of $R$ that are also disjoint from $X$, i.e., $S \neq \emptyset$, for all $Y \in S$ we have $Y \subseteq R$ and for all $Y, Z \in S \cup \{X\}$ we have $Y \cap Z = \emptyset$

- $R$-relation $r$ satisfies $X : \{Y_1, \ldots, Y_k\}$ on $R$ iff for all $t_1, \ldots, t_{k+1} \in r$ the following condition is satisfied: if $t_i[X] = t_j[X]$ for all $1 \leq i, j \leq k + 1$, then there is some $t \in r$ such that $t[XY_i] = t_i[XY_i]$ for $i = 1, \ldots, k$ and $t[X(R - XY_1 \cdots Y_k)] = t_{k+1}[X(R - XY_1 \cdots Y_k)]$

- Let $X, Y_1, \ldots, Y_k \subseteq R$ be pairwise disjoint and $k \geq 1$. An $R$-relation $r$ satisfies the FHD $X : \{Y_1, \ldots, Y_k\}$ on $R$ if and only if $r = r[XY_1] \bowtie \cdots \bowtie r[XY_k] \bowtie r[X(R - XY_1 \cdots Y_k)]$. 
Sound Inference Rules for FDs and FHDs

- **Reflexivity (\(R_F\)):**
  \[
  X \rightarrow Y \quad \quad Y \subseteq X
  \]

- **Extension (\(E_F\)):**
  \[
  X \rightarrow XY
  \]

- **Transitivity (\(T_F\)):**
  \[
  X \rightarrow Z
  \]

- **Augmentation (\(A_H\)):**
  \[
  XZ : \{Y_1 - Z, \ldots, Y_k - Z\}
  \]

- **Transitivity (\(T_H\)):**
  \[
  X : \{Y_1, \ldots, Y_k\}
  \]

- **Implication (\(I_{FH}\)):**
  \[
  X \rightarrow Y
  \]

- **Mixed Subset (\(S_{FH}\)):**
  \[
  X \rightarrow Y \cap Z \quad Y \cap Z = \emptyset
  \]

- **Subset (\(S_H\)):**
  \[
  X \rightarrow Y \cap Z \quad Y \cap Z = \emptyset
  \]

- **Mixed Transitivity (\(T_{FH}\)):**
  \[
  X \rightarrow Y \cap Z \quad Y \cap Z = \emptyset
  \]
Main Results for FDs and FHDs

\[
\mathcal{HAC}_C = \mathcal{HAC} \cup \{S_H, M_H\} = (\mathcal{HC}_C - \{T_{FH}\}) \cup \{S_{FH}\}
\]

appropriate: adequate and complementary

\[
\mathcal{HAC}_C = (\mathcal{HF}_C - \{T_{FH}\}) \cup \{S_{FH}\}
\]

adequate

\[
\mathcal{HC}_C = \mathcal{HF}_C \cup \{S_H, M_H\}
\]

complementary

\[
\mathcal{HF}_C = \{R_F, E_F, T_F, A_H, T_H, O_H, I_{FH}, T_{FH}, C^R_H\}
\]
Example Transformation - Shifting Complementation

• inappropriate inference:

\[
\begin{align*}
X &: \{Y\} \\
C^R_H &: X : \{R - XY\} \quad X(R - XY): \{Y_1, \ldots, Y_k\} \\
T_H &: X : \{Y_1, \ldots, Y_k, R - XY\}
\end{align*}
\]

• appropriate inference: applying subset rule \(S_H\) instead

\[
\begin{align*}
X &: \{Y\} \\
S_H &: X : \{Y_1 \cap Y, \ldots, Y_k \cap Y, Y - Y_1 \cdots Y_k\} \\
C^R_H &: X : \{Y_1, \ldots, Y_k, R - XY_1 \cdots Y_k(Y - Y_1 \cdots Y_k)\} \\
= R - XYZ &= R - XY
\end{align*}
\]

• \(Y\) and \(X(R - XY)\) are disjoint, and \(Y_1 \cdots Y_k\) and \(X(R - XY)\) are disjoint; hence, \(Y_1 \cdots Y_k \subseteq Y\), and thus \(Y_i \cap Y = Y_i\) for all \(i = 1, \ldots, k\)
Example Transformation - Eliminating Complementation

- inappropriate inference:

\[
\begin{align*}
X & : \{Y\} \\
C^R_H : & \quad X : \{R - XY\} \quad X(R - XY) \to Z \\
T_{FH} : & \quad X \to Z - X(R - XY) = Y \cap Z
\end{align*}
\]

- notice that \(X\) and \(Y\) are disjoint, Hence, \(X(R - XY) = R - Y\) and, consequently, \(Z - (R - Y) = Y \cap Z\)

- appropriate inference:

\[
\begin{align*}
X & : \{Y\} \quad X(R - XY) \to Z \\
S_{FH} : & \quad X \to Y \cap Z
\end{align*}
\]


Literature

- Beeri, Fagin, Howard: A Complete Axiomatization for Functional and Multivalued Dependencies in Database Relations, SIGMOD, pp. 47-61, 1977
Conclusion and Outlook

• Complementation rule is mere means of database normalisation:
  • FHD-inference requires at most one application in last step;
  • FD-inference does not require an application at all

• Inappropriate inferences can be converted into appropriate inferences.

• Study the significance of undetermined universes for views!

• Find appropriate axiomatisations of F(H)Ds in ER Models.

• Find appropriate axiomatisations of F(H)Ds in Nested Models.

• Find appropriate axiomatisations of fuzzy and approximate F(H)Ds.