When Data Dependencies over SQL Tables Meet the Logics of Paradox and S-3

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Main contributions

▶ unifying theory of three orthogonal classes of data dependencies

▶ duality results between the:
  ← implication of data dependency classes and
  ← implication of propositional para-consistent fragments

▶ insight into SQL’s NOT NULL constraint as:
  ← an effective control mechanism to
  ← balance expressiveness with efficiency of entailment relations
Running example - Informal

suppliers deliver articles from a location at a certain cost:

CREATE TABLE SUPPLIES (  
    Article CHAR[20],  
    Supplier VARCHAR NOT NULL,  
    Location VARCHAR NOT NULL,  
    Cost CHAR[8]);

DBMS explicitly enforces additional business rules:  
for every article there is at most one supplier,  
cost determined by article and location, and  
location sets determined by supplier (independent of article & cost)

are the following business rules already enforced implicitly?  
cost is determined by article  
location sets determined by article (independent of supplier & cost)
Running Example - Formal

- relation schema $R$:
  $$\text{SUPPLIES}=\{\text{Article}, \text{Supplier}, \text{Location}, \text{Cost}\}$$

- no-information null value $\text{ni}$ (distinguished value of each domain)
  $\leftarrow$ value may not exist, or value exists and is unknown [Zaniolo]

- null-free subschema NFS $R_s \subseteq R$ over $R$ [Atzeni/Morfuni]:
  $$\text{SUPPLIES}_s=\{\text{Supplier, Location}\}$$
  $\leftarrow$ $r$ satisfies $R_s$ iff every $t \in r$ is $R_s$-total ($\forall A \in R_s(t(A) \neq \text{ni})$)

- data dependencies enforce business rules:
  $\leftarrow$ FDs: $A \rightarrow S$ and $AL \rightarrow C$
  $\leftarrow$ MVD: $S \rightarrow L$
Data Dependencies in Presence of No-information Nulls

relation $r$ satisfies FD $X \rightarrow Y$ over $R$ [Lien, Atzeni/Morfuni]:

$\iff$ if for all tuples $t_1, t_2 \in r$ we have:

if $t_1(X) = t_2(X)$ and $t_1, t_2$ are $X$-total, then $t_1(Y) = t_2(Y)$

relation $r$ satisfies MVD $X \rightarrow Y$ over $R$ [Lien]:

$\iff$ if for all tuples $t_1, t_2 \in r$ we have:

if $t_1(X) = t_2(X)$ and $t_1, t_2$ are $X$-total, then there is some tuple $t$ in $r$ such that $t(XY) = t_1(XY)$ and $t(X(R - Y)) = t_2(X(R - Y))$

relationship to decompositions [Lien]:

$\iff$ if $r$ satisfies $X \rightarrow Y$, then $r_X[R] = r_X[XY] \bowtie r_X[X(R - Y)]$

$\iff$ $r$ satisfies $X \rightarrow Y$ if and only if $r_X[R] = r_X[XY] \bowtie r_X[X(R - Y)]$
The Implication Problem

Implication problem:
\[ \text{input: } R, \text{ NFS } R_s \text{ and FD/MVD set } \Sigma \cup \{ \varphi \} \text{ over } R \]
\[ \text{output: } \begin{cases} \text{yes, if } \Sigma \models R_s \varphi & \text{where } \Sigma \models R_s \varphi \text{ holds, if} \\ \text{no, if } \Sigma \nmodels R_s \varphi \end{cases} \]

\[ R = ASLC, \ R_s = SL, \ \Sigma = \{ A \rightarrow S; AL \rightarrow C; S \rightarrow L \}: \]
\[ \Sigma \models R_s A \rightarrow C \text{ and } \Sigma \models R_s A \rightarrow L \]

If \( R_s = ALC \) instead, then \( \Sigma \nmodels R_s A \rightarrow C \) and \( \Sigma \nmodels R_s A \rightarrow L \):

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</table>
Outline

► Motivation
  ← Previous work
  ← Applications

► Axiomatization and Efficient Algorithms

► Equivalences to Para-consistent Logics
  ← FDs, MVDs and NOT NULL constraints
  ← Boolean dependencies and NOT NULL constraints
  ← Applications

► Conclusion and Open Problems
Motivation
Previous Work in this Research Area

► over total relations (every attribute is NOT NULL)
  ← Fagin, Beeri, Howard: axiomatization of FDs and MVDs
  ← Beeri, then Sagiv, then Galil: almost linear-time algorithms
  ← Fagin: FD equivalence to Boolean propositional Horn clauses
  ← Sagiv, Delobel, Fagin, Stott Parker jr: extension to MVDs and BDs

► over partial relations (no attribute is NOT NULL)
  ← Lien: axiomatization of FDs and MVDs

► for arbitrary null-free subschemata:
  ← Atzeni and Morfuni: axiomatization for FDs only
  ← Atzeni and Morfuni: linear-time algorithm

► our theory unifies these three orthogonal theories
Application 1:
Decompositions for Efficient Processing of Updates

\( R = ASLC, \Sigma = \{ A \rightarrow S; AL \rightarrow C; S \rightarrow L \} \):

\[ R_1 = SL; R_2 = AS \] and \( R_3 = AC \)

properties of decomposition:

\( R_s = ASL \): both lossless and faithful
\( \Sigma \models_{ASL} A \rightarrow C \)

\( R_s = AS \): lossless, not faithful
\( \Sigma \not\models_{AS} A \rightarrow C, AL \rightarrow C \) lost

\( R_s = SL \): not lossless, faithful
\( \Sigma \models_{SL} A \rightarrow C \)

\( R_s = S \): neither lossless nor faithful
Application 2: Rewriting for Efficient Processing of Queries

\[ R = ASLC, \quad R_s = SL, \quad \Sigma = \{ A \rightarrow S; \ AL \rightarrow C; \ S \rightarrow L \} \]:

- retrieve locations and costs associated with same articles:
  
  \[
  \text{SELECT } R.L, \ R'.C \\ \text{FROM } R, \ R \text{ AS } R' \\ \text{WHERE } R.A = R'.A
  \]

- since \( \Sigma \models_{R_s} A \rightarrow C \) we can rewrite into
  
  \[
  \text{SELECT } R.L, \ R.C \text{ FROM } R
  \]
Application 3: Inference Attacks

- attacker may infer secrets without violating access control policies
- $R = ASLC, R_S = SL, \Sigma = \{A \rightarrow S; AL \rightarrow C; S \rightarrow L\}$:
- some users prohibited access to
  $$\Psi = (\exists X_S) R(Kiwi, X_S, Wellington, 2NZD)$$
- attacker asks the following queries without violating access policy $\Psi$:
  - $\Phi_1 = (\exists X_S)(\exists X_L) R(Kiwi, X_S, X_L, 2NZD)$
  - $\Phi_2 = (\exists X_S)(\exists X_C) R(Kiwi, X_S, Wellington, X_C)$
- attacker exploits fact that $\Sigma \models_{SL} A \rightarrow L$ holds:
  - applying $A \rightarrow L$ to $\Phi_1$ and $\Phi_2$ reveal $\Phi$
  - if $S \not\in R_S$, attacker cannot draw that conclusion
- understanding entailment assists in preventing inference attacks
Axiomatization
and
Efficient Algorithms
The FD/MVD Axiomatization $\mathcal{D}$ for arbitrary NFS $R_s$

\begin{align*}
\frac{XY \rightarrow Y}{\text{reflexivity, } R_F} \\
\frac{X \rightarrow Y}{\text{decomposition, } D_F} \\
\frac{X \rightarrow Y; X \rightarrow Z}{\text{FD union, } U_F}
\end{align*}

\begin{align*}
\frac{X \rightarrow Y}{\text{R-complementation, } C^R_M} \\
\frac{X \rightarrow Y; X \rightarrow Z}{\text{MVD union, } U_M} \\
\frac{X \rightarrow W; Y \rightarrow Z}{Y \subseteq X(W \cap R_s)}
\end{align*}

\begin{align*}
\frac{X \rightarrow W; Y \rightarrow Z}{Y \subseteq X(W \cap R_s)}
\end{align*}
The Dependency Basis and Attribute Closure

- Let $Dep_{\Sigma,R_s}(X) := \{Y \mid \Sigma \vdash_\Box X \rightarrow Y\}$

- $(Dep_{\Sigma,R_s}(X), \subseteq, \cup, \cap, (\cdot)^C, \emptyset, R)$ is a finite Boolean algebra

- $DepB_{\Sigma,R_s}(X)$ is the set of all atoms of $(Dep_{\Sigma,R_s}(X), \subseteq, \emptyset)$

- $DepB_{\Sigma,R_s}(X)$ is called the dependency basis of $X$ wrt $\Sigma$ and $R_s$

- $X^+_{\Sigma,R_s} = \{A \mid \Sigma \vdash_\Box X \rightarrow A\}$ is the closure of $X$ wrt $\Sigma$ and $R_s$

- Let $\Sigma$ be an FD/MVD set and $R_s$ an NFS over $R$. Then we have:

  $\Sigma \vdash_\Box X \rightarrow Y$ iff $Y = \bigcup \mathcal{Y}$ for some $\mathcal{Y} \subseteq DepB_{\Sigma,R_s}(X)$,

  $\Sigma \vdash_\Box X \rightarrow Y$ iff $Y \subseteq X^+_{\Sigma,R_s}$, and

  if $\Sigma \vdash_\Box X \rightarrow A$, then $\{A\} \in DepB_{\Sigma,R_s}(X)$
Sketch of the Completeness Proof

Given: FD/MVD set $\Sigma \cup \{\varphi\}$ and NFS $R_s$ over $R$

$\leftarrow$ show: if $\Sigma \models_{R_s} \varphi$, then we can infer $\varphi$ from $\Sigma$ by $\mathcal{D}$

For $\varphi = X \rightarrow Y \notin \Sigma^+$ find $r_\varphi$ to violate $X \rightarrow Y$ and satisfy $\Sigma$ & $R_s$

$\leftarrow$ $\text{DepB}_{\Sigma,R_s}(X) = \{\{A\} \mid A \in X_{\Sigma,R_s}^+\} \cup \{W_1, \ldots, W_k\}$

$\leftarrow$ since $\varphi \notin \Sigma^+$: $Y$ is not union of elements in $\text{DepB}_{\Sigma,R_s}(X)$

$\leftarrow$ there is $i \in \{1, \ldots, k\}$ such that $Y \cap W_i \neq \emptyset$ and $Y - W_i \neq \emptyset$

$\leftarrow$ following relation $r_\varphi := \{t, t'\}$ satisfies $\Sigma$ and $R_s$, but violates $\varphi$

<table>
<thead>
<tr>
<th>$X(X_{\Sigma,R_s}^+ \cap R_s)$</th>
<th>$(X_{\Sigma,R_s}^+ - X) - R_s$</th>
<th>$W_1 \cap R_s$</th>
<th>$W_1 - R_s$</th>
<th>$\cdots$</th>
<th>$W_i$</th>
<th>$\cdots$</th>
<th>$W_k \cap R_s$</th>
<th>$W_k - R_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>0\ldots0</td>
<td>ni\ldotsni</td>
<td>0\ldots0</td>
<td>ni\ldotsni</td>
<td>0\ldots0</td>
<td>0\ldots0</td>
<td>ni\ldotsni</td>
<td></td>
</tr>
<tr>
<td>$t'$</td>
<td>0\ldots0</td>
<td>ni\ldotsni</td>
<td>0\ldots0</td>
<td>ni\ldotsni</td>
<td>1\ldots1</td>
<td>0\ldots0</td>
<td>ni\ldotsni</td>
<td></td>
</tr>
</tbody>
</table>

Let $\varphi$ denote FD $X \rightarrow Y \notin \Sigma^+$

$\leftarrow$ FD union rule: there is $A \in Y$ such that $X \rightarrow A \notin \Sigma^+$

$\leftarrow$ hence, $A \notin X_{\Sigma,R_s}^+$, e.g. $A \in W_i$

$\leftarrow$ again, $r_\varphi$ satisfies $\Sigma$ and $R_s$, but violates $\varphi$
Computing the Dependency Basis

- extend Beeri’s $O(|\Sigma|^4)$ algorithm to compute $DepB_{\Sigma,R}(X)$ from total case by extending Beeri’s rule to arbitrary NFS $R_s$:

  $\begin{align*}
  X &\rightarrow W; Y \rightarrow Z \\
  X &\rightarrow W \cap Z; X \rightarrow W - Z
  \end{align*}$

  $W \cap Y = \emptyset; Y \subseteq XR_s$

- Galil implemented idea in $O(|\Sigma| + \min\{k_\Sigma, \log p_\Sigma\} \times |\Sigma|)$ time

  $\leftarrow k_\Sigma$ the number of MVDs in $\Sigma$, $p_\Sigma$ the cardinality of $DepB_{\Sigma,R}(X)$

  $\leftarrow$ algorithm runs in linear time when $\Sigma$ contains only FDs

- let $\Sigma[U]$ contain elements of $\Sigma$ whose left-hand side is subset of $U$

  $\leftarrow X^+_{\Sigma,R_s} = X^+_{\Sigma[XR_s],R_s}$ and $DepB_{\Sigma,R_s}(X) = DepB_{\Sigma[XR_s],R_s}(X)$

  $\leftarrow$ only need $Y \rightarrow Z$ or $Y \rightarrow Z \in \Sigma[XR_s]$ in general case

- yields an $O(|\Sigma| + \min\{k_{\Sigma[XR_s]}, \log p_{\Sigma[XR_s]}\} \times |\Sigma[XR_s]|)$

  time algorithm to compute $DepB_{\Sigma,R_s}(X)$ under the NFS $R_s$
An Efficient Algorithm for Deciding the Implication Problem

- if \( \text{DepB}_{\Sigma,R_s}(X) \) known, the implication problem \( \Sigma \models_{R_s} \varphi \) for a given FD or MVD \( \varphi \) with left-hand side \( X \) can be decided in linear time

- in particular, \( \Sigma \models_{R_s} X \rightarrow A \) holds when \( \{A\} \in \text{DepB}_{\Sigma,R_s}(X) \) and \( \Sigma[X R_s] \) contains an FD \( Y \rightarrow Z \) with \( A \in Z \)

- in practice, a minor modification of Galil’s algorithm gives an algorithm for deciding \( \Sigma \models_{R_s} \varphi \)

\[
O(|\Sigma| + \min\{k_{\Sigma[X R_s]}, \log \bar{p}_{\Sigma[X R_s]}\} \times |\Sigma[X R_s]|)
\]

\( \bar{p}_\Sigma \) is the number of sets in \( \text{DepB}_{\Sigma,R_s}(X) \) that have non-empty intersection with the right-hand side of \( \varphi \)
Equivalences to Para-consistent Logics
Some Intuition for the Equivalences

the relation

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obeys the FDs $A \rightarrow L$ and $L \rightarrow C$, but it violates the FD $A \rightarrow C$

therefore, the two tuples in the relation simultaneously

agree on $Location$ (to satisfy the FD $A \rightarrow L$) and

disagree on $Location$ (to satisfy the FD $L \rightarrow C$)

this is a paradox: $L'$ interpreted as $true$ and $false$ at the same time

reasoning about data dependencies in the presence of nulls seems to correspond to reasoning in some para-consistent logic

we will pinpoint this correspondence

paradox resolved by distinguishing LHSs and RHSs of dependencies
A Para-consistent Logic

- proof-theoretic aim: non-explosive reasoning under inconsistencies
  - inconsistent theories can have models (in contrast to classical logic)

- in our para-consistent logic $LP_{R_s}$ a sentence can be either
  - true (and not false) $T$, or false (and not true) $F$, or
  - paradoxical (both true and false) $P$

- this yields a three-valued logic based on Kleene’s truth tables:

<table>
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<tr>
<th></th>
<th>¬</th>
<th>∧TPF</th>
<th>∨TPF</th>
<th>→TPF</th>
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<tr>
<td>$TF$</td>
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<td>$PP$</td>
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- $LP_{R_s}$ has $\{P, T\}$ as its set of designated truth values
  - variables corresponding to attributes in $R_s$ interpreted as $T$ or $F$
  - $LP_{R_s}$-interpretation and entailment $|= LP_{R_s}$ defined as usual
we define the $LP_{R_s}$-formulae that correspond to FDs and MVDs

$\phi : R \rightarrow \mathcal{L}_R$ bijection between

$\rightarrow$ relation schema $R$ and

$\rightarrow$ propositional variable set $\mathcal{L}_R = \{ A' \mid A \in R \}$

$\rightarrow$ paradox-free variable set: $\mathcal{L}_{R_s} = \{ A' \mid A \in R_s \}$

extend $\phi$ to mapping $\Phi$ from FD/MVD set over $R$ to $\mathcal{L}_R^*$:

$\rightarrow \Phi(X \rightarrow B) = (\bigwedge_{A \in X} A') \rightarrow B'$

$\rightarrow \Phi(X \rightarrow Y) = (\bigwedge_{A \in X} A') \rightarrow ((\bigwedge_{B \in Y-X} B') \lor (\bigwedge_{C \in R-XY} C'))$

disjunctions over zero disjuncts are $\top$

conjunctions over zero conjuncts are $\top$

denote $\Phi(\varphi)$ as $\varphi'$ and $\Phi(\Sigma) = \{ \Phi(\sigma) \mid \sigma \in \Sigma \} = \Sigma'$
Our Equivalence

\textbf{Theorem:} Let $\Sigma \cup \{\varphi\}$ be a set of FDs and MVDs, and let $R_s$ be an NFS over $R$. Let $\Sigma' \cup \{\varphi'\}$ denote the set of corresponding $LP_{R_s}$-formulae over $\mathcal{L}_R$. Then:

$$\Sigma \models_{R_s} \varphi \quad \text{if and only if} \quad \Sigma' \models_{LP_{R_s}} \varphi'.$$

- $R_s = \emptyset$: Logic of Paradox; generally: equivalence to $S$-3 with $S = \mathcal{L}_{R_s}$
- $R = ASLC$, $R_s = ALC$, $\Sigma = \{A \rightarrow S; AL \rightarrow C; S \rightarrow L\}$, $r$

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shows that $\Sigma$ implies neither $\varphi_1 = A \rightarrow L$ nor $\varphi_2 = A \rightarrow C$

\textbf{counter-example relations} $r = \{t, t'\}$ and interpretations $\omega'_r$:

\begin{itemize}
  \item for $\omega'_r$ we get $\omega'_r(A') = \top$, $\omega'_r(S') = \Pi$, $\omega'_r(L') = \bot$ and $\omega'_r(C') = \bot$
  \item $\omega'_r$ is a model of $\Sigma'$ but not a model of neither $\varphi'_1$ nor $\varphi'_2$
\end{itemize}
Proof Arguments

(1) two-tuple relations suffice to decide $\Sigma \models_{R_s} \varphi$

(2) $\{t, t'\}$ satisfies $\varphi$ if and only if $\omega'_{\{t,t'\}}$ is an $LP_{R_s}$-model of $\varphi'$ with

$\hookrightarrow$ special $LP_{R_s}$ interpretation for two tuples $t, t'$ over $R$:

$$\omega'_{\{t,t'\}}(A') = \begin{cases} 
\top, & \text{if } ni \neq t(A) = t'(A) \neq ni \\
\bot, & \text{if } t(A) = ni = t'(A) \\
F, & \text{if } t(A) \neq t'(A)
\end{cases}$$

$\hookrightarrow$ for $A' \in L_{R_s}$: $\omega'_{\{t,t'\}}(A') = \top$ iff $t(A) = t'(A)$

$\hookrightarrow$ Fagin’s special interpretation in case of NOT NULL attribute

(3) NOT NULL attributes correspond to paradox-free variables:

$\hookrightarrow$ if $\{t, t'\}$ satisfies $R_s$, then $\omega'_{\{t,t'\}}$ is an $LP_{R_s}$-interpretation

$\hookrightarrow$ for $LP_{R_s}$-interpret. $\omega'$, $\models_{\{t,t'\}} R_s$ and $\omega' = \omega'_{\{t,t'\}}$ for some $\{t, t'\}$
Application: Boolean Dependencies

- equivalance holds when duplicate tuples permitted to occur

<table>
<thead>
<tr>
<th>$\Sigma$</th>
<th>$\varphi$</th>
<th>$\Sigma’\models\varphi$ (Combined)</th>
<th>$\Sigma_0\models\varphi$ (Expression)</th>
<th>$\Sigma’\models\varphi_0$ (“Data”)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Any</td>
<td>Any</td>
<td>coNP-complete</td>
<td>coNP-complete</td>
<td>$\mathcal{O}(</td>
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<td>Any</td>
<td>CNF</td>
<td>$\mathcal{O}(</td>
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<td>Any</td>
<td>DNF</td>
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Monotonicity: if $R_s \subseteq R_s' \subseteq R$ and $\Sigma \models_{R_s} \varphi$, then $\Sigma \models_{R_s'} \varphi$

Uniformity: $\Sigma \models_{R_s} \varphi$ decidable in $\mathcal{O} \left( |\Sigma| \times |\varphi| \times 2^{R_s} \right)$ time
Conclusion
and
Open Problems
The big Picture

PL = LP_{A,B,C}

LP_{A,B} \quad LP_{A,C} \quad LP_{B,C}

LP_{A} \quad LP_{B} \quad LP_{C}

LP = LP_{\phi}
Conclusion

► SQL tables permit arbitrary NOT NULL constraints
► constraints arising in practice require at least FDs and MVDs
► previous orthogonal theories did not accommodate these requirements
► new theory subsumes all previous ones & retains nice properties:
  ↪ finite axiomatization
  ↪ implication problem in almost linear time
  ↪ logical foundation by (para-consistent) logics
  ↪ also: cannot be extended to other constraints (provably)
► hence: gain in expressiveness, no loss in efficiency
► NOT NULL constraints: an effective mechanism for designers to balance expressiveness with efficiency of entailment relations
► new upper time bounds for para-consistent entailment relations
Research Agenda

- investigate other null values, at least:
  - → not applicable
  - → does not exist
  - → exists, but unknown (theory valid under possible world semantics)

- identify the corresponding logics

- study normalization and its justification

- automate semantic query optimization with constraints

- investigate the properties of Armstrong tables

- probability theory: correspondences to conditional independencies

- develop preference-based specifications of constraints